Entropy of a Kerr–de Sitter black hole due to arbitrary spin fields

Shuang-Qing Wu*
Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

Mu-Lin Yan†
CCAST (World Lab), P.O. Box 8730, Beijing 100080, People's Republic of China
and Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

(Received 20 March 2003; revised manuscript received 2 September 2003; published 27 February 2004)

The Newman-Penrose formalism is used to derive the Teukolsky master equations controlling massless scalar, neutrino, electromagnetic, gravitino, and gravitational field perturbations of the Kerr–de Sitter spacetime. Then the quantum entropy of a nonextreme Kerr–de Sitter black hole due to arbitrary spin fields is calculated by the improved thin-layer brick wall model. It is shown that the subleading order contribution to the entropy is dependent on the square of the spins of particles and that of the specific angular momentum of black holes as well as the cosmological constant. The logarithmic correction of the spins of particles to the entropy relies on the rotation of the black hole and the effect of the cosmological constant.

DOI: 10.1103/PhysRevD.69.044019 PACS number(s): 04.70.Dy, 04.62.+v, 97.60.Lf

I. INTRODUCTION

Ever since Bekenstein [1] and Hawking [2] discovered that black holes are thermodynamic objects endowed with a temperature proportional to its surface gravity and an entropy equal to one fourth of its surface area, much effort has been devoted to investigating the statistical [3], quantum [4], or dynamic [5] origin of black hole entropy [6]. A proposal for the study of statistical origin of the black hole entropy is the brick wall method (BWM) suggested by 't Hooft [7]. In this model, the black hole entropy is identified with the statistical-mechanical entropy of a thermal gas of quantum field excitations outside the event horizon, which is composed of the leading order correction (i.e., the standard Bekenstein-Hawking entropy) and the logarithmic contributions to the black hole entropy. The subleading order corrections contain two parts, in general: the logarithmic term from the integral in the optical space and the logarithmic correction from the effective potential. This method, especially with the aid of the Newman-Penrose formalism [8,9], has been successfully used in studies of the statistical-mechanical entropy of scalar fields for some static black holes [7,10] and stationary axisymmetric black holes [11,12]. In the case of various static spherically symmetric black holes, it has also been applied to evaluate the entropy of spinor fields [13,14] and electromagnetic fields [15]. Recently the BWM has been used to calculate the entropy of Dirac field for the Kerr(-Newman) black hole [16]. However all these calculations did not consider the logarithmic contribution from the coupling of the spin of particles with the rotation of black holes. This kind of spin-rotation coupling effect appears [17] in the Hawking thermal radiation spectrum of Dirac particles and photons in some nonstationary Kerr(-Newman) black holes [18]. Though the subleading order correction had been included in Ref. [14], but only the logarithmic contribution from the integral in the optical space had been considered there, the logarithmic term from the effective potential including the quadratic spin terms still had been neglected in those studies. The latter correction cannot be ignored in general, because it is of the same order as the former in the high frequency approximations. The presence of a logarithmic divergence in the entropy of the quantum scalar field has also been confirmed by other different approaches [19–21], such as the conical singularity approach, the heat kernel expansion method, and the ζ-function regularization technique, etc. As far as the rotating black hole case is concerned, the logarithmic contribution both from the integral in the dragged optical space and from the effective potential had been considered by using the BWM only for the scalar field in recent research [22].

Recently much attention has been paid to the quantum entropies of black holes due to higher spin fields [23–26]. Li, Shen and Gao, and Gao and Shen [23], investigated the entropies of arbitrary spin fields in various spherically symmetric black holes, but did not consider the logarithmic contribution from the integral in the optical space nor the subleading order correction from the effective potential including the quadratic spin terms. Li [24] studied the entropy of one component of massless fields with spin $s=1/2$, 1, and 2 in the Reissner-Nordström black hole by the BWM and found that the logarithmic correction to the entropy depends on the linear term of the spins of the particles. Jing and Yan [25] calculated the entropy up to subleading terms of massless fields of spin $s>0$ for the Kerr black hole and showed that the contribution of the spins to the logarithmic terms shall decrease the statistical-mechanical entropy of a Kerr black hole. López-Ortega [26] extended this analysis to the Rarita-Schwinger field case but pointed out that the entropy is increased by the logarithmic terms relating to the square of spins of particles. However the coefficients of the quadratic spin terms in the expressions of entropy presented in Refs. [25] and [26] are incompatible with each other and their

*E-mail address: sqwu@ustc.edu.cn
†E-mail address: mlyan@ustc.edu.cn

validity is in doubt. Thus it deserves to take the logarithmic terms to the black hole entropy from the effective potential into account in details. How the spins of the quantum field changes the quantum entropy of a rotating black hole is an interesting question and needs to be further clarified. The knowledge of the entropy as a function of the spin of the field will be helpful to study the species dependence problem of the BWM on a rotating black hole.

On the other hand, although the original BWM has contributed a great deal to the understanding and calculation of the entropy of a black hole, there are some drawbacks in it such as the little mass approximation and taking the term including L^3 (L being the ''infrared cutoff'') as a contribution of the vacuum surrounding the black hole, etc. The model is constructed on the basis of thermal equilibrium at a large scale, so it cannot be applied to cases out of equilibrium, such as spacetime with two horizons, for example, a Schwarzschild–de Sitter black hole and Vaidya black hole [27,28]. However one can improve this original BWM by taking only the entropy of a thin layer near the event horizon of a black hole into account, and utilize this improved thin-layer brick wall method to resolve some thermal nonequilibrium problems that can hardly be treated by the original BWM [27,28].

For a Kerr–de Sitter spacetime with nondegenerate horizons, it has a cosmological horizon and an outer black hole event horizon. As shown in Refs. [29] and [30], there will appear two cases: (1) the general case that the temperatures of these two horizons are distinct. In this case, a nonextreme Kerr–de Sitter spacetime is a thermal nonequilibrium system. When the cosmological constant is very small, the two horizons will separate far away. Then each horizon, in principle, can be treated as an isolated thermodynamical system. Although the total system consisted of the two horizons is thermal nonequilibrium, the thin layer near the horizon can be taken as a local thermal equilibrium system. The quantum entropy of such a black hole can be calculated via this improved BWM which means that the entropy comes from a thin layer near the horizon. By arguments on thermodynamics of a composite system, the total entropy of a nonextreme Kerr–de Sitter spacetime is then taken the sum of the contribution from each horizon; and (2) a special case that the temperatures of the two distinct horizons are equal. The metric in this special case is called the lukewarm solution [29,30]. The lukewarm solution achieves global thermodynamic equilibrium, however it is unstable to changes in the mass of the black hole. In the lukewarm case, the notion of local thermal equilibrium is therefore not required, and one can still work with the original BWM to calculate the entropy of each horizon. It should be pointed out that the expression of the entropy in the lukewarm case is just a special case of that obtained in our general considerations. So we shall mainly consider the general case and leave the lukewarm case for a special discussion.

In this paper, the entropies of nonextreme Kerr–de Sitter black holes due to higher spin fields are taken up for consideration on which the effect of the cosmological constant and that of the spins of particles are emphasized. The subject is important because the study of spacetimes which are asymptotically de Sitter has received a great deal of attention recently. The recent astrophysical observations [31] of type Ia supernovae which indicate a positive cosmological constant [32] and the recent dS/conformal field theory correspondence [33] are two main motivations for studying the thermodynamics of black holes with a cosmological constant. It should be noted that a realistic black hole may be in an asymptotically nonflat space, thus it becomes important to investigate the effect of the cosmological constant on the entropies of these kinds of black holes. In Ref. [34], the renormalized black hole entropy for the massive scalar field in a Schwarzschild-anti–de Sitter space has been considered recently by Winstanley via the original ''brick wall'' method. Although recent work [35] has dealt with the entropy due to massless Dirac fermionic and scalar fields in the Newman-Unti-Tomborino Kerr-Newman–de Sitter spacetime case, it puts emphasis on the improved brick wall model and only cares for the leading correction to black hole entropy, i.e., the standard Bekenstein-Hawking entropy.

The purpose of this paper is to deduce expressions of the entropy of nonextreme Kerr–de Sitter black holes arising from arbitrary spin fields by using the improved brick-wall method and to investigate effects of the spins of particles and the cosmological constant on the statistical entropy. In this study, we carefully deal with the subleading order contribution to the entropy not only from the integral in the dragged optical space but also the logarithmic term from the effective potential including the quadratic spin terms, namely, the subleading corrections to the entropy arising from the coupling of the spins of the particles with the rotation of the black holes and the cosmological constant, regardless of a positive or negative one.

The paper is organized as follows. Within the Newman-Penrose formalism [8,9], we derive in Sec. II the master equations governing massless Klein-Gordon scalar, Weyl neutrino, Maxwell electromagnetic, Rarita-Schwinger gravitino, and linearized Einstein gravitational field perturbations of the Kerr–de Sitter space. Section III is devoted to deducing expressions of the statistical-mechanical entropy of the nonextreme Kerr–de Sitter black hole due to arbitrary spin fields by using the thin-layer BWM. In Sec. IV, we give some arguments about the difference with previously published results. The last section summarizes our discussions. Appendix A separates the Teukolsky master equations and presents the Teukolsky-Starobinsky identities in the Kerr–de Sitter geometry. In Appendix B, we calculate some integrals by the thin-layer BWM.

II. PERTURBATIONS OF SPIN FIELDS IN THE KERR–DE SITTER SPACE

The line element of the Kerr–de Sitter spacetime can be written in a coordinate system of Boyer-Lindquist type as [36,37]

$$ds^2 = -\frac{\Delta_r}{\chi^2\Sigma}(dt - a\sin^2\theta\, d\varphi)^2 + \frac{\Delta_\theta \sin^2\theta}{\chi^2\Sigma}[a\,dt - (r^2+a^2)d\varphi]^2 + \Sigma\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right), \tag{1}$$

where

$$\Delta_r=(r^2+a^2)\left(1-\frac{r^2}{l^2}\right)-2Mr,$$

$$\Delta_\theta=1+\frac{a^2}{l^2}\cos^2\theta,$$

$$\chi=1+\frac{a^2}{l^2},$$

$$\Sigma=\rho\rho^*=r^2+a^2\cos^2\theta,\quad \rho=r+ia\cos\theta,$$

$$\rho^*=r-ia\cos\theta.$$

Here M is the mass of the black hole, a is its angular momentum per unit mass, and $\tilde{\Lambda}=3/l^2$ is the cosmological constant. To fit with the recent astrophysical observations [31] of type Ia supernovae, we require that the constant $\tilde{\Lambda}$ is positive but very small [32]. This means that the radius of the de Sitter horizon is very large. The metric determinant, Ricci scalar, and the only nonvanishing Weyl scalar are, $\sqrt{-g}=\Sigma\sin\theta/\chi^2$, $\mathcal{R}=4\tilde{\Lambda}=12/l^2$, $\tilde{\Psi}_2=-M/\rho^{*3}$, respectively.

The metric has coordinate singularities at the roots of $\Delta_r=0$. For a range of parameters that satisfy the condition $l^2/3\gg M^2>a^2$, there are four distinct real roots, three of which are positive and correspond (in decreasing order) to the cosmological, outer, and inner horizons of the black hole, respectively. The fourth root is negative and nonphysical. A significant case is the lukewarm solution [29,30] which satisfies the condition $M^2=a^2\chi^2$. In this special case the horizons are generally distinct whereas the surface gravity on both horizons is identical. In the allowed range of the physically meaningful parameters a and M, the admissible classes of the Kerr–de Sitter solutions are categorized completely by Booth and Mann [30] in details. In this paper, we shall assume that the horizons are nondegenerate and are mainly interested in the general case that the temperature of the cosmological horizon and that of the outer black hole event horizon are different from one another, and separate the lukewarm case for a special discussion.

To derive a single master equation governing the perturbations of the Kerr–de Sitter spacetime, we work it within the Newman-Penrose formalism [8,9] by choosing the null-tetrad vectors as

$$l^\mu=\frac{1}{\Delta_r}[(r^2+a^2)\chi,\Delta_r,0,a\chi],$$

$$n^\mu=\frac{1}{2\Sigma}[(r^2+a^2)\chi,-\Delta_r,0,a\chi],$$

$$m^\mu=\frac{1}{\sqrt{2\Delta_\theta}\rho}\left(i\chi a\sin\theta,0,\Delta_\theta,\frac{i\chi}{\sin\theta}\right),$$

$$\bar{m}^\mu=(m^\mu)^*, \tag{2}$$

and obtain the nonvanishing spin coefficients as follows:

$$\tilde{\rho}=\frac{-1}{\rho^*},\quad \mu=\frac{-\Delta_r}{2\Sigma\rho^*},\quad \gamma=\mu+\frac{\Delta_r'}{4\Sigma},$$

$$\tau=\frac{-ia\sqrt{\Delta_\theta}\sin\theta}{\sqrt{2}\Sigma},$$

$$\pi=\frac{ia\sqrt{\Delta_\theta}\sin\theta}{\sqrt{2}\rho^{*2}},\quad \beta=\frac{\sqrt{\Delta_\theta}}{2\sqrt{2}\rho}\left(\cot\theta+\frac{\Delta_\theta'}{2\Delta_\theta}\right),\quad \alpha=\pi-\beta^*, \tag{3}$$

where a prime denotes the partial differential with respect to its argument.

Assuming that the azimuthal and time dependence of the perturbed fields will be of the form $e^{i(m\varphi-\omega t)}$, we find that the directional derivatives are [37]

$$D=l^\mu\partial_\mu=\mathcal{D}_0,\quad \Delta=n^\mu\partial_\mu=\frac{-\Delta_r}{2\Sigma}\mathcal{D}_0^\dagger,$$

$$\delta=m^\mu\partial_\mu=\frac{\sqrt{\Delta_\theta}}{\sqrt{2}\rho}\mathcal{L}_0^\dagger,\quad \bar{\delta}=\bar{m}^\mu\partial_\mu=\frac{\sqrt{\Delta_\theta}}{\sqrt{2}\rho^*}\mathcal{L}_0, \tag{4}$$

where

$$\mathcal{D}_n=\frac{\partial}{\partial r}-\frac{i\chi K_1}{\Delta_r}+n\frac{\Delta_r'}{\Delta_r},$$

$$\mathcal{L}_n=\frac{\partial}{\partial\theta}-\frac{\chi K_2}{\Delta_\theta}+n\left(\cot\theta+\frac{\Delta_\theta'}{2\Delta_\theta}\right),$$

$$\mathcal{D}_n^\dagger=\frac{\partial}{\partial r}+\frac{i\chi K_1}{\Delta_r}+n\frac{\Delta_r'}{\Delta_r},$$

$$\mathcal{L}_n^\dagger=\frac{\partial}{\partial\theta}+\frac{\chi K_2}{\Delta_\theta}+n\left(\cot\theta+\frac{\Delta_\theta'}{2\Delta_\theta}\right),$$

and

$$K_1=\omega(r^2+a^2)-ma,\quad K_2=a\omega\sin\theta-\frac{m}{\sin\theta} \tag{5}$$

with the relations

$$K_1-K_2a\sin\theta=\omega\Sigma,\qquad K_2'+K_2\cot\theta=2a\omega\cos\theta. \tag{6}$$

Using the Newman-Penrose formalism [8,9] it can be shown that perturbation master equations in the Kerr–de Sitter geometry are separable for massless spin $s=0$, 1/2, 1, 3/2, and 2 fields [38]. Teukolsky's master equations [39] controlling the perturbations of Kerr-de Sitter black hole for massless arbitrary spin fields ($s=1/2$, 1, 3/2, and 2 for Weyl neutrino, source-free Maxwell electromagnetic, Rarita-Schwinger gravitino, and the linearized Einstein gravitational fields, respectively) read [40]

$$\{[D-(2s-1)\epsilon+\epsilon^*-2s\tilde{\rho}-\tilde{\rho}^*](\Delta-2s\gamma+\mu) -[\delta-(2s-1)\beta-\alpha^*-2s\tau+\pi^*](\bar{\delta}-2s\alpha+\pi) -(s-1)(2s-1)\tilde{\Psi}_2\}\Phi_s=0, \tag{7}$$

for spin weight $s=1/2$, 1, 3/2, 2 and

$$\{[\Delta+(2s-1)\gamma-\gamma^*+2s\mu+\mu^*](D+2s\epsilon-\tilde{\rho}) -[\bar{\delta}+(2s-1)\alpha+\beta^*+2s\pi-\tau^*](\delta+2s\beta-\tau) -(s-1)(2s-1)\tilde{\Psi}_2\}\Phi_{-s}=0, \tag{8}$$

for spin weight $s=-1/2,-1,-3/2,-2$ (s is the spin of the perturbed fields). It is clear that Eqs. (7) and (8) are also valid when $s=0$, they coincide with the massless conformally coupled scalar field equation

$$(\Box+\mathcal{R}/6)\Phi=0, \tag{9}$$

with $\Phi=\Phi_0=\Phi_{-0}$.

All the above equations are separable by using the Newman-Penrose formalism, and can be written as (ignoring the factor $e^{i(m\varphi-\omega t)}$)

$$\left[\frac{1}{\Sigma}(\Delta_r\mathcal{D}_1\mathcal{D}_s^\dagger+\sqrt{\Delta_\theta}\mathcal{L}_{1-s}^\dagger\sqrt{\Delta_\theta}\mathcal{L}_s) +2(2s-1)\left(\frac{i\chi\omega}{\rho^*}-\frac{s-1}{l^2}\right)\right]\Phi_s=0, \tag{10}$$

$$\left[\frac{1}{\Sigma}(\Delta_r\mathcal{D}_{1-s}^\dagger\mathcal{D}_0+\sqrt{\Delta_\theta}\mathcal{L}_{1-s}\sqrt{\Delta_\theta}\mathcal{L}_s^\dagger) -2(2s-1)\left(\frac{i\chi\omega}{\rho^*}+\frac{s-1}{l^2}\right)\right](\rho^{*2s}\Phi_{-s})=0. \tag{11}$$

It can be directly shown that Eqs. (10) and (11) are also satisfied by the scalar Debye potentials $\phi_s=\Phi_s/\rho^{*2s}$ and $\phi_{-s}=\rho^{*2s}\Phi_{-s}$, which obey [40]

$$\{[D-(2s-1)\epsilon+\epsilon^*-\tilde{\rho}^*][\Delta-2s\gamma-(2s-1)\mu] -[\delta-(2s-1)\beta-\alpha^*+\pi^*][\bar{\delta}-2s\alpha-(2s-1)\pi] -(s-1)(2s-1)\tilde{\Psi}_2\}\phi_s=0, \tag{12}$$

$$\{[\Delta+(2s-1)\gamma-\gamma^*+\mu^*][D+2s\epsilon+(2s-1)\tilde{\rho}] -[\bar{\delta}+(2s-1)\alpha+\beta^*-\tau^*][\delta+2s\beta+(2s-1)\tau] -(s-1)(2s-1)\tilde{\Psi}_2\}\phi_{-s}=0. \tag{13}$$

In Ref. [38], the authors showed that Eqs. (10) and (11) can be further separated and transformed into Heun equations. In the case of a Kerr black hole, they degenerate to the generalized spheroidal wave equation [41,42], a confluent form of Heun equation [42]. Exact solutions to these equations, and integral equations as well as other related applications can be found in Refs. [38,41–44]. To the end of this paper, we do not take these into account, but just present in Appendix A the radial Teukolsky-Starobinsky identities [40,45] where the coefficient C_2 corrects the previously published results [46]. From their obvious expressions of the above Eqs. (10) and (11)

$$\frac{1}{\Sigma}\left\{\Delta_r^{-s}\frac{\partial}{\partial r}\left(\Delta_r^{1+s}\frac{\partial}{\partial r}\right)+\frac{\chi^2K_1^2-is\chi K_1\Delta_r'}{\Delta_r}+\frac{s}{2}\Delta_r''+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\Delta_\theta\sin\theta\frac{\partial}{\partial\theta}\right)-\frac{1}{\Delta_\theta}\left[\chi K_2-s\left(\frac{\Delta_\theta'}{2}+\Delta_\theta\cot\theta\right)\right]^2 +4is\chi\omega\rho-\frac{4s^2+2}{l^2}\Sigma\right\}\Phi_s=0, \tag{14}$$

and

$$\frac{1}{\Sigma}\left\{\Delta_r^{s}\frac{\partial}{\partial r}\left(\Delta_r^{1-s}\frac{\partial}{\partial r}\right)+\frac{\chi^2K_1^2+is\chi K_1\Delta_r'}{\Delta_r}-\frac{s}{2}\Delta_r''+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\Delta_\theta\sin\theta\frac{\partial}{\partial\theta}\right)-\frac{1}{\Delta_\theta}\left[\chi K_2+s\left(\frac{\Delta_\theta'}{2}+\Delta_\theta\cot\theta\right)\right]^2 -4is\chi\omega\rho-\frac{4s^2+2}{l^2}\Sigma\right\}\phi_{-s}=0, \tag{15}$$

one can easily find that they are dual by interchanging $s\to -s$. Thus one only needs to consider the case of positive spin state $p=s$, and obtain the results for the negative spin state $p=-s$ by substituting $s\to -s$. Equations (14) and (15) can be combined into the form of Teukolsky's master equation [39]

$$\left\{\frac{\Delta_r}{\Sigma}\frac{\partial^2}{\partial r^2}+\frac{(1+s)\Delta_r'}{\Sigma}\frac{\partial}{\partial r}+\frac{\Delta_\theta}{\Sigma}\frac{\partial^2}{\partial\theta^2}+\frac{\Delta_\theta'+\Delta_\theta\cot\theta}{\Sigma}\frac{\partial}{\partial\theta}+\frac{\omega^2\chi^2}{\Sigma}\left[\frac{(r^2+a^2)^2}{\Delta_r}-\frac{a^2\sin^2\theta}{\Delta_\theta}\right]\right.$$
$$-\frac{2\omega ma\chi^2}{\Sigma}\left(\frac{r^2+a^2}{\Delta_r}-\frac{1}{\Delta_\theta}\right)+\frac{m^2\chi^2}{\Sigma}\left(\frac{a^2}{\Delta_r}-\frac{1}{\Delta_\theta\sin^2\theta}\right)+\frac{2s\omega\chi}{\Sigma}\left[a\sin\theta\left(\frac{\Delta_\theta'}{2\Delta_\theta}-\cot\theta\right)-\frac{i\Delta_r'}{2\Delta_r}(r^2+a^2)+2ir\right]$$
$$\left.+\frac{2sm\chi}{\Sigma}\left[\frac{ia\Delta_r'}{2\Delta_r}-\frac{1}{\sin\theta}\left(\frac{\Delta_\theta'}{2\Delta_\theta}+\cot\theta\right)\right]+\frac{s}{2\Sigma}\Delta_r''-\frac{4s^2+2}{l^2}-\frac{s^2\Delta_\theta}{\Sigma}\left(\frac{\Delta_\theta'}{2\Delta_\theta}+\cot\theta\right)^2\right\}\Psi_s=0,$$
$$(s=0,\pm 1/2,\pm 1,\pm 3/2,\pm 2). \tag{16}$$

In the above master Equation (16), one can identify the term proportional to $(4s^2+2)/l^2=(2s^2+1)\mathcal{R}/6$ with the conformally coupled term for arbitrary spin fields in virtue of the nonvanishing Ricci scalar in the Kerr–de Sitter spacetime. In the next section we will utilize this equation to obtain the density of states by using the Wentzel-Kramers-Brillouin (WKB) approximation scheme.

III. ENTROPY OF KERR–de SITTER BLACK HOLES DUE TO SPIN FIELDS

Now we calculate the entropy due to arbitrary spin fields for the nonextreme Kerr–de Sitter black hole by the thin-layer BWM. First we try to seek the total number of modes with energy less than ω. In order to do this, we make use of the WKB approximation and substitute $\Psi_s\sim e^{iG(r,\theta)}$ into the above Teukolsky's master Equation (16), then we obtain

$$\frac{\Delta_r}{\Sigma}k_r^2+\frac{\Delta_\theta}{\Sigma}k_\theta^2+\frac{\omega^2\chi^2}{\Sigma}\left[\frac{a^2\sin^2\theta}{\Delta_\theta}-\frac{(r^2+a^2)^2}{\Delta_r}\right]+\frac{2\omega ma\chi^2}{\Sigma}\left(\frac{r^2+a^2}{\Delta_r}-\frac{1}{\Delta_\theta}\right)+\frac{m^2\chi^2}{\Sigma}\left(\frac{1}{\Delta_\theta\sin^2\theta}-\frac{a^2}{\Delta_r}\right)$$
$$+\frac{2s\omega\chi a\sin\theta}{\Sigma}\left(\cot\theta-\frac{\Delta_\theta'}{2\Delta_\theta}\right)+\frac{2sm\chi}{\Sigma\sin\theta}\left(\frac{\Delta_\theta'}{2\Delta_\theta}+\cot\theta\right)+\frac{s^2\Delta_\theta}{\Sigma}\left(\frac{\Delta_\theta'}{2\Delta_\theta}+\cot\theta\right)^2+\frac{4s^2+2}{l^2}-\frac{s}{2\Sigma}\Delta_r''=0, \tag{17}$$

in which, $k_r=G_{,r}$ and $k_\theta=G_{,\theta}$ are the "wave numbers." In terms of the covariant metric components $g_{\mu\nu}$, Eq. (17) can be rewritten as

$$\frac{k_r^2}{g_{rr}}+\frac{k_\theta^2}{g_{\theta\theta}}+\frac{g_{\varphi\varphi}\omega^2+2g_{t\varphi}m\omega+g_{tt}m^2}{\mathcal{D}}$$
$$+2(\omega B+mC)+H_s=0, \tag{18}$$

where

$$g_{rr}=\frac{\Sigma}{\Delta_r},\qquad g_{\theta\theta}=\frac{\Sigma}{\Delta_\theta},\qquad g_{tt}=\frac{\Delta_\theta a^2\sin^2\theta-\Delta_r}{\chi^2\Sigma},$$

$$g_{t\varphi}=\frac{\Delta_r-(r^2+a^2)\Delta_\theta}{\chi^2\Sigma}a\sin^2\theta,$$

$$g_{\varphi\varphi}=\frac{(r^2+a^2)^2\Delta_\theta-\Delta_r a^2\sin^2\theta}{\chi^2\Sigma}\sin^2\theta,$$

$$\mathcal{D}=g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2=-\frac{\Delta_r\Delta_\theta\sin^2\theta}{\chi^4}, \tag{19}$$

$$B=\frac{s\chi a\sin\theta}{\Sigma}\left(\cot\theta-\frac{\Delta_\theta'}{2\Delta_\theta}\right),$$

$$C=\frac{s\chi}{\Sigma\sin\theta}\left(\frac{\Delta_\theta'}{2\Delta_\theta}+\cot\theta\right),$$

$$H_s=\frac{s^2\Delta_\theta}{\Sigma}\left(\frac{\Delta_\theta'}{2\Delta_\theta}+\cot\theta\right)^2+\frac{4s^2+2}{l^2}-\frac{s}{2\Sigma}\Delta_r''.$$

To obtain the density of states, let us suppose that the quantum field is rotating with an angular velocity Ω_h in a thin-layer very near the horizon of the nonextreme Kerr–de Sitter black hole. Substituting $\mathcal{E}=\omega-m\Omega_h$ into Eq. (18), we reduce it into the form

$$\frac{k_r^2}{g_{rr}}+\frac{k_\theta^2}{g_{\theta\theta}}+\frac{g_{\varphi\varphi}\mathcal{E}^2+2(g_{t\varphi}+g_{\varphi\varphi}\Omega_h)m\mathcal{E}+\tilde{g}_{tt}m^2}{\mathcal{D}}$$
$$+2[\mathcal{E}B+m(\Omega_hB+C)]+H_s=0, \tag{20}$$

and then rewrite it as

$$\frac{k_r^2}{g_{rr}}+\frac{k_\theta^2}{g_{\theta\theta}}+\frac{-\tilde{g}_{tt}}{-\mathcal{D}}(m+m_0)^2=\frac{1}{-\tilde{g}_{tt}}(\mathcal{E}+sW)^2-V_s, \tag{21}$$

where

$$\tilde{g}_{tt}=g_{tt}+2g_{t\varphi}\Omega_h+g_{\varphi\varphi}\Omega_h^2,$$

$$m_0=\frac{(g_{t\varphi}+g_{\varphi\varphi}\Omega_h)\mathcal{E}+(\Omega_hB+C)\mathcal{D}}{g_{tt}+2g_{t\varphi}\Omega_h+g_{\varphi\varphi}\Omega_h^2}, \tag{22}$$

$$sW=(g_{tt}+g_{t\varphi}\Omega_h)B-(g_{t\varphi}+g_{\varphi\varphi}\Omega_h)C,$$

$$V_s=H_s-(g_{tt}B^2-2g_{t\varphi}BC+g_{\varphi\varphi}C^2)=P-\frac{s\Delta_r''}{2\Sigma}.$$

$\tilde{g}_{tt}$ is the temporal component of the metric of the dragged optical space [47,48], W is the angular velocity caused by the rotation of the black hole and can be called the "spin potential," while V_s is the effective potential [11] which is equal to $\mu^2+\xi\mathcal{R}=\mu^2+12\xi/l^2$ in the case of a massive scalar field with an arbitrary conformally coupling ($\xi=1/6$ for the minimally coupling), where μ is the mass of the field.

For a given energy $\omega=\mathcal{E}+m\Omega_h$ and a given azimuthal angular momentum m, Eq. (21) represents an ellipsoid of three-dimensional momentum space, *a compact surface*, spanned by k_r, k_θ, and m, which is a subspace of a six-dimensional phase space, supposed that the following conditions could be satisfied:

$$g_{rr}>0, \qquad g_{\theta\theta}>0, \qquad -\tilde{g}_{tt}>0, \qquad -\mathcal{D}>0,$$

$$(\mathcal{E}+sW)^2+\tilde{g}_{tt}V_s\geqslant 0. \tag{23}$$

Therefore in this case, for the positive spin state of spin fields the number of modes with $\mathcal{E}$ is equal to the number of states in this classical phase space [11]

$$\Gamma(\mathcal{E},s)=\frac{1}{(2\pi)^3}\int drd\theta d\varphi\int dk_rdk_\theta dm$$
$$=\frac{1}{3\pi}\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{(-\tilde{g}_{tt})^2}[(\mathcal{E}+sW)^2+\tilde{g}_{tt}V_s]^{3/2}. \tag{24}$$

Here we impose the improved thin-layer BWM boundary conditions in the integral with respect to the radial coordinate r, that is, we assume that the quantum field is equal to zero for $r\leqslant r_h+\varepsilon$ and $r\geqslant r_h+N\varepsilon$, with $r_h\gg\varepsilon$. The ultraviolet cutoff ε is a small distance from the horizon r_h to the inner brick wall, the cutoff parameter N is a sufficient big integer to remove the infrared divergence. In other words, the infrared cutoff L in the original BWM is now replaced by $r_h+N\varepsilon$ in the upper limit of the radial integral. This reflects a significant difference from the original BWM, and such improved BWM is called the thin-layer model [28], it can overcome some defects in the original BWM.

In the original brick-wall model, it is supposed that the black hole is in thermal equilibrium with the external field in a large spatial region. This method cannot be applied to a nonequilibrium system such as a system of spacetime with two horizons, for example, Schwarzschild–de Sitter spacetime and Vaidya spacetime [27,28]. In such spacetimes, two problems arise: (a) the thermal equilibrium between the external field and the hole is unstable, so the thermal equilibrium on a large scale basis for the brick wall does not exist; and (b) since the two horizons have different temperatures, there exists no global thermal equilibrium over the entire spacetime, and statistical physics laws are invalid there.

In the thin-layer method, the entropy of the black hole is mainly attributed to the degrees of the freedom of the field in the thin-layer ($r_h+\varepsilon\leqslant r\leqslant r_h+N\varepsilon$) covering the surface of the horizon. The Bekenstein-Hawking entropy should be associated with the fields in this small region near the horizon, where the local thermal equilibrium exists and statistical physics laws are still valid. To guarantee that the notion of local thermal equilibrium can work very well, here one supposes that the physical quantities of the thermodynamic properties of the exciting field outside the hole vary slightly in the vicinity of the horizon. On the one hand, the length of the thin-layer region near the horizon must be small enough on a macroscopic scale so that the physical quantities in the region can approximately be treated as some constants in the vicinity, and approximate equilibrium in the small region is achieved. On the other hand, the region must be large enough on a microscopic scale so that the statistical mechanics are valid for the fields in the near horizon region, and the thermodynamic variables can be defined through a partition function. In order for local equilibrium to be maintained, it is necessary that the hole's radiation is slight enough that the fluctuation of thermodynamic properties of fields can be treated as a small quantity. In most cases, this condition can be satisfied except for those ones at the Planckian scale.

In the spacetime that has two horizons with two different temperatures, there exists no global thermal equilibrium in the entire spacetime, however approximate thermal equilibrium exists in the two layers near each horizon. Thus the global thermal equilibrium is not needed, the validity of local thermal equilibrium is crucial to the discussion [28]. In such a thin-layer BWM, the total entropy is mainly attributed to the two thin-layers near the two horizons, namely, it is a linear sum of the area of each horizon.

In the case of a nonextreme Kerr–de Sitter black hole with nondegenerate horizons, situation is more involved. The issue of local versus global thermal equilibrium is a rather delicate one. As mentioned before, there are two cases that can happen: (1) In the general case, the temperatures of the cosmological and black hole event horizons are distinct; and (2) in the lukewarm case, both temperatures are equal. In the general, nonextreme Kerr–de Sitter background, there is no time-like Killing vector which is well-defined in the whole region surrounded by the two horizons. Hence the system in the general case cannot be in thermal equilibrium globally, namely, there exists no thermal equilibrium in the entire spacetime since the two horizons have different temperatures. In addition, there cannot exist a global thermal equilibrium between the external field and the hole in a large spatial region. Thus in this general case, one must work with the thin-layer method to calculate the entropy of the Kerr–de Sitter black hole. In the lukewarm case ($M^2=a^2\chi^2$), however, the cosmological and black hole event horizons are in thermal equilibrium (but unstable to changes in the mass of the black hole [29]). In this special case, one need not introduce the notion of local thermal equilibrium; one can still adopt the original BWM to calculate the entropy of each horizon. The main interest of this paper is in the general case, and we will turn to the lukewarm case for a special consideration. In both cases, the total entropy of a quantum field in the nonextreme Kerr–de Sitter spacetime is summed up from the entropies corresponding to the maximal and minimal spin-weight components.

Summing over the positive and negative spin states $p=\pm s$, we get the total states number

$$\Gamma(\mathcal{E})=\frac{g_s}{2}[\Gamma(\mathcal{E},s)+\Gamma(\mathcal{E},-s)]$$

$$=\frac{g_s}{6\pi}\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{(-\tilde{g}_{tt})^2}\{[(\mathcal{E}+sW)^2+\tilde{g}_{tt}V_s]^{3/2}+[(\mathcal{E}-sW)^2+\tilde{g}_{tt}V_{-s}]^{3/2}\} \tag{25}$$

$$\approx\frac{g_s}{3\pi}\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{(-\tilde{g}_{tt})^2}\times\left[\mathcal{E}^3+3\left(\frac{1}{2}\tilde{g}_{tt}P+s^2W^2\right)\mathcal{E}-\frac{3s^2\Delta_r''}{4\Sigma}\tilde{g}_{tt}W\right]$$

$$\equiv\frac{g_s}{3\pi}[I_1\mathcal{E}^3+3(I_2+s^2I_3)\mathcal{E}-3s^2I_4].$$

In the above, we have expanded Eq. (25) in the high frequency approximation and introduced an appropriate degeneracy g_s for each species of particles (it is well-known that $g_s=2s+1$ in the nonrelativistic quantum statistics case). Here $g_s=1$ for scalar field $(s=0)$, $g_s=2$ for Weyl neutrino $(s=1/2)$, Maxwell electromagnetic $(s=1)$, Rarita-Schwinger gravitino $(s=3/2)$ and linearized Einstein gravitational $(s=2)$ fields, and $g_s=4$ for massless Dirac field $(s=1/2)$, respectively. The following four integrals in terms of the thin-layer BWM are calculated in Appendix B:

$$I_1=\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{\tilde{g}_{tt}^2},$$

$$I_2=\frac{1}{2}\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{\tilde{g}_{tt}}P,$$

$$I_3=\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{\tilde{g}_{tt}^2}W^2,$$

$$I_4=\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{\tilde{g}_{tt}}\frac{\Delta_r''}{4\Sigma}W, \tag{26}$$

$$P=\frac{4s^2+2}{l^2}+\frac{s^2\Delta_\theta}{\Sigma}\left(\frac{\Delta_\theta'}{2\Delta_\theta}+\cot\theta\right)^2-(g_{tt}B^2-2g_{t\varphi}BC+g_{\varphi\varphi}C^2).$$

Apparently the integral I_1 is related to the volume of the dragged optical space [47,48].

It is known that a "physical space" must be dragged by the gravitational field with an azimuthal angular velocity Ω_h in the stationary rotating spacetime. Since we have supposed that a quantum field in a thin-layer very near the horizon is in local thermal equilibrium with the nonextreme Kerr–de Sitter black hole at the asymptotic temperature $1/\beta$ measured by an observer located at the spatial infinity, it is appropriate to assume that the quantum field is rotating with angular velocity Ω_h in this thin-layer also. Note that here we use the asymptotic quantities rather than those measured by a local (corotating) observer in the thin-layer. In fact the equivalence principle implies that a thin-layer system in local thermal equilibrium has a local Tolman inverse temperature given by $\beta_{\rm local}=\beta\sqrt{-\tilde{g}_{tt}}$, with β being the asymptotic inverse temperature [49]. The temperature measured by this local observer is a local temperature $\beta_{\rm local}$, correspondingly he also measures a local (blue-shifted) energy $\omega_{\rm local}=\omega/\sqrt{-\tilde{g}_{tt}}$, where the energy ω measured by the asymptotic observer is associated with the coordinate time t. The quantity $\beta\omega=\beta_{\rm local}\omega_{\rm local}$ is an invariant due to the first thermodynamic law, and so is $\beta(\omega-m\Omega_h)$, etc. As the equivalence between using local quantities and using asymptotic ones has already been proven in Ref. [50], one can directly adopt the asymptotic quantities, needless to make further conversions. For such a local (quasi-)equilibrium ensemble of states of spin fields, the free energy can be expressed as follows:

$$F=\frac{(-1)^{-2s}}{\beta}\int dm\int d\omega g(\omega,m)\times\ln[1-(-1)^{2s}e^{-\beta(\omega-m\Omega_h)}]$$

$$=-\int dm\int d\omega\frac{\Gamma(\omega,m)}{e^{\beta(\omega-m\Omega_h)}-(-1)^{2s}}$$

$$=-\int_0^\infty d\mathcal{E}\frac{\Gamma(\mathcal{E})}{e^{\beta\mathcal{E}}-(-1)^{2s}}, \tag{27}$$

where we have made a variable substitution $\mathcal{E}=\omega-m\Omega_h$. The formulas for the density of states $g(\omega,m)=d\Gamma(\omega,m)/d\omega$ and $\Gamma(\mathcal{E})=\int\Gamma(\mathcal{E}+m\Omega_h,m)dm$ had also been used. Inserting Eq. (25) into Eq. (27) and then carrying out the integral over $\mathcal{E}$, we get the expression for the free energy

$$F=-\frac{g_s}{6\pi}\int d\theta\int_{r_h+\varepsilon}^{r_h+N\varepsilon}dr\frac{\sqrt{-g}}{(-\tilde{g}_{tt})^2}\int_0^\infty\frac{d\mathcal{E}}{e^{\beta\mathcal{E}}-(-1)^{2s}}\times\{[(\mathcal{E}+sW)^2+\tilde{g}_{tt}V_s]^{3/2}+[(\mathcal{E}-sW)^2+\tilde{g}_{tt}V_{-s}]^{3/2}\}$$
$$\approx-\frac{g_s}{3\pi}\int_0^\infty\frac{d\mathcal{E}}{e^{\beta\mathcal{E}}-(-1)^{2s}}[I_1\mathcal{E}^3+3(I_2+s^2I_3)\mathcal{E}-3s^2I_4]$$
$$=-g_s\left[2\zeta(4)\frac{15+(-1)^{2s}}{16\pi\beta^4}I_1+\zeta(2)\frac{3+(-1)^{2s}}{4\pi\beta^2}(I_2+s^2I_3)-\zeta(1)\frac{1+(-1)^{2s}}{2\pi\beta}s^2I_4\right], \tag{28}$$

where $\zeta(n)=\Sigma_{k=1}^{\infty}1/k^n$ is the Riemann zeta function, $\zeta(4)=\pi^4/90$, $\zeta(2)=\pi^2/6$, etc.

We are now ready to obtain the entropy of the nonextreme Kerr–de Sitter black hole due to arbitrary spin fields from the standard formula $S=\beta^2(\partial F/\partial\beta)$,

$$S=\frac{g_s}{2\pi}\left[\zeta(4)\frac{15+(-1)^{2s}}{\beta^3}I_1+\zeta(2)\frac{3+(-1)^{2s}}{\beta}(I_2+s^2I_3)-\zeta(1)(1+(-1)^{2s})s^2I_4\right]. \tag{29}$$

Next, we are in a position to consider the four integrals $I_1\sim I_4$. It is easy to obtain the Hawking temperature and the area corresponding to the horizon r_h of the nonextreme Kerr–de Sitter black hole

$$\beta_h^{-1}=\frac{\kappa_h}{2\pi}=\frac{\Delta'_{r_h}}{4\pi\chi(r_h^2+a^2)}=\frac{\Delta'_{r_h}}{\chi^2A_h},\qquad A_h=4\pi(r_h^2+a^2)/\chi. \tag{30}$$

By means of the thin-layer BWM, we take the angular velocity of a quantum field near the horizon of the nonextreme Kerr–de Sitter black hole as $\Omega_h=a/(r_h^2+a^2)$, and find that only the integrals I_1 and I_2 contribute to the leading and subleading terms in the entropy, while the integrals I_3 and I_4 can be ignored as usual because the integral I_3 can be attributed to the contribution of the vacuum surrounding the hole due to $I_3\sim O(\epsilon)$ and I_4 vanishes at least up to the order $O(\epsilon)$. The final expressions for these integrals are presented in Appendix B where the ultraviolet cutoff ε is replaced by the proper distance η from the horizon to the inner brick wall $\eta=\int_{r_h}^{r_h+\varepsilon}\sqrt{g_{rr}}dr\approx2(\varepsilon\Sigma_h/\Delta'_{r_h})^{1/2}$. In order to be comparable with the results already appeared in the literature and to simplify the expression, we have set the new ultraviolet cutoff ϵ and infrared cutoff Λ by $\eta^2=2\epsilon^2/15$ and $N=\Lambda^2/\epsilon^2$ as did in Refs. [20,25] and [26]. With utilization of the results given by Eq. (B7), we obtain the statistical-mechanical entropy

$$S/g_s=\pi^3\frac{15+(-1)^{2s}}{180\beta^3}I_1+\pi\frac{3+(-1)^{2s}}{12\beta}I_2$$
$$=\frac{15+(-1)^{2s}}{90\chi(\beta\kappa_h/\pi)^3}\left[\frac{15(r_h^2+a^2)}{4\epsilon^2}+\left(1-\frac{3r_h^2+a^2}{2l^2}\right)\ln\frac{\Lambda}{\epsilon}\right]+\frac{3+(-1)^{2s}}{24\chi(\beta\kappa_h/\pi)}\left\{-\frac{4(r_h^2+a^2)}{l^2}+s^2\left[\frac{a^2-r_h^2}{r_h^2}+\frac{r_h^2-a^2}{l^2}+\left(\frac{r_h^2+a^2}{r_h^2}-\frac{9r_h^2+a^2}{l^2}\right)\frac{r_h^2+a^2}{ar_h}\arctan\left(\frac{a}{r_h}\right)\right]\right\}\ln\frac{\Lambda}{\epsilon}. \tag{31}$$

Assuming that the field is in the Hartle-Hawking vacuum state and taking $\beta=\beta_h$, we get that the entropy is given by

$$S/g_s=\frac{15+(-1)^{2s}}{16}\left[\frac{A_h}{48\pi\epsilon^2}+\frac{1}{45\chi}\left(1-\frac{3r_h^2+a^2}{2l^2}\right)\ln\frac{\Lambda}{\epsilon}\right]+\frac{3+(-1)^{2s}}{4}\left\{-\frac{A_h}{12\pi l^2}+\frac{s^2}{12\chi}\left[\frac{a^2-r_h^2}{r_h^2}+\frac{r_h^2-a^2}{l^2}+\left(\frac{r_h^2+a^2}{r_h^2}-\frac{9r_h^2+a^2}{l^2}\right)\frac{r_h^2+a^2}{ar_h}\arctan\left(\frac{a}{r_h}\right)\right]\right\}\ln\frac{\Lambda}{\epsilon}. \tag{32}$$

Equations (31) and (32) show that the entropy of a nonextreme Kerr–de Sitter black hole due to arbitrary spin fields consists of two parts, the leading order contribution and the subleading order corrections, or equivalently the contribution from the integral I_1 in the dragged optical space and the logarithmic term I_2 from the effective potential including the quadratic spin terms. The logarithmic corrections consist of the one from the integral I_1 and that from the integral I_2, both of them are of the same order, therefore the latter cannot be thrown away any more. It should be noted that the coefficients of the logarithmic divergence is universal (invariant under a change in the value of the cutoff, or even under a change in the regulator scheme). The expression of Eq. (32) may settle down the species dependence problem of the brick wall entropy on a rotating black hole, it covers many previously obtained results.

To see what role the conformally coupling plays in the calculation, we also present the expression of the entropy for a massive scalar field in the nonextreme Kerr-Newman–de Sitter black hole

$$S=\frac{A_h}{48\pi\epsilon^2}+\frac{1}{45\chi}\left[1-\frac{3r_h^2+a^2}{2l^2}-\frac{3Q^2}{4r_h^2}\right.$$
$$\left.\times\left(1+\frac{r_h^2+a^2}{ar_h}\arctan\left(\frac{a}{r_h}\right)\right)\right]\ln\frac{\Lambda}{\epsilon}$$
$$-\frac{A_h}{24\pi}\left(\mu^2+\frac{12\xi}{l^2}\right)\ln\frac{\Lambda}{\epsilon}, \tag{33}$$

with an arbitrary conformally coupling constant ξ. It is not difficult to find that the conformally coupling will contribute a logarithmic correction to the entropy. By setting $N=1$ in Eqs. (32) and (33), we recover the standard Bekenstein-Hawking entropy.

IV. DISCUSSIONS

The final result that we obtain for the entropies of arbitrary spin fields in the nonextreme Kerr–de Sitter space deserves some remarks.

(a) The entropies given above have summed up the contribution from the maximal and minimal spin-weight states of a quantum field. Under the condition that satisfies $l^2/3 \gg M^2 > a^2$, the cosmological horizon separates far away from the outer black hole horizon, the total entropy of the nonextreme Kerr–de Sitter black hole can take a linear sum of the area of the two horizons.

The calculations here are valid both for the outer black hole event horizon case and for the cosmological horizon case because the two horizons are in an equal position. We think it is also valid for the black hole event horizon of the Kerr-anti–de Sitter space by changing the sign of the cosmological constant $\tilde{\Lambda}$.

(b) The entropies depend not only on the spins of the particles but also on the cosmological constant except different spin fields obey different statistics. They rely on the quadratic terms of s^2 and $-1/l^2$ as well as a^2.

(c) Both the contribution of the spins and that of the cosmological constant to the entropies are in subleading order. The spins have a tendency to increase the entropies, but the effect of a positive cosmological constant tends to decrease them.

(d) The logarithmic term from the spins of the particles not only depends on the spin-rotation coupling effect but also on the coupling between the spins of particles and the cosmological constant. Figure 1 shows that how the coefficient of the square term of the spins for the logarithmic correction to the entropies is affected by the specific angular momentum of the hole and the cosmological constant.

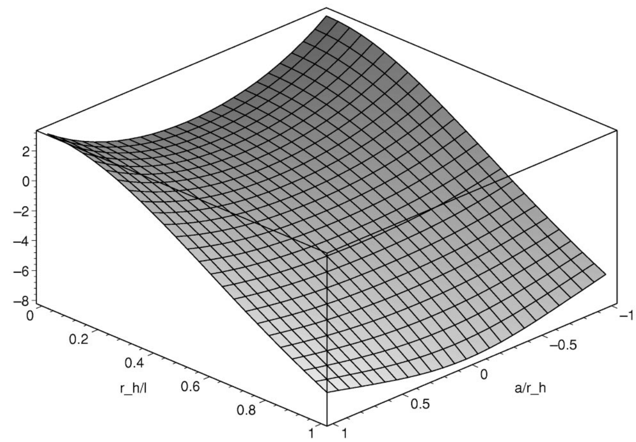

FIG. 1. The coefficient of the quadratic spin term for the logarithmic correction to the entropies is dependent on the specific angular momentum of the hole and the cosmological constant. The entropy increases when $|a/r_h|$ becomes large and cuts down when r_h/l increases.

(e) Three special cases may be very interesting:

Case I: In the Schwarzschild–de Sitter black hole case, the specific angular momentum vanishes and the black hole is nonrotating. The improved thin-layer model still works in this case, then the entropy due to arbitrary spin fields is

$$S/g_s=\frac{15+(-1)^{2s}}{16}\left[\frac{A_h}{48\pi\epsilon^2}+\frac{1}{45}\left(1-\frac{3r_h^2}{2l^2}\right)\ln\frac{\Lambda}{\epsilon}\right]$$
$$-\frac{3+(-1)^{2s}}{4}\,\frac{1+2s^2}{12\pi l^2}A_h\ln\frac{\Lambda}{\epsilon}, \tag{34}$$

where $A_h=4\pi r_h^2$.

It is not difficult to find that the entropy still depends on the square of the spins of particles unless in the Schwarzschild and Reissner-Nordström black hole cases. The existence of a positive cosmological constant will decrease the entropy whereas the effect of a negative one tends to increase it, vice versa. This result is in accordance with the entropy of a massive scalar field for the Schwarzschild-anti–de Sitter black hole by using the original BWM in Ref. [34] where a negative cosmological constant is assumed and if the square mass of the scalar field μ^2 is replaced by $2/l^2$.

Case II: In the Kerr black hole case, the cosmological constant $\tilde{\Lambda}$ goes to zero (or in the $l\to\infty$ limit), the improved thin-layer model degenerates to the original BWM by simply setting the infrared cutoff $L=N\varepsilon$ without any change in the ultraviolet cutoff ϵ. By the equality $N=\Lambda^2/\epsilon^2$ and the relation $\varepsilon\approx\eta^2\simeq\epsilon^2$, the infrared cutoff $L\approx N\epsilon^2\simeq\Lambda^2$ is consistent with the definition $\Lambda^2=L\epsilon^2/\varepsilon$ as given in Refs. [20,25] and [26]. (There is a minor difference in the definition of the infrared cutoff L because N can be sufficient large, here it is a shift by r_h from the original infrared cutoff introduced by 't Hooft [7].) It should be noted that the system in the brick wall region is, in fact, still in a local thermal equilibrium, and the thin-layer model still works very well. Taking these into account, the expression of the brick wall entropy in the Kerr black hole case reduces to

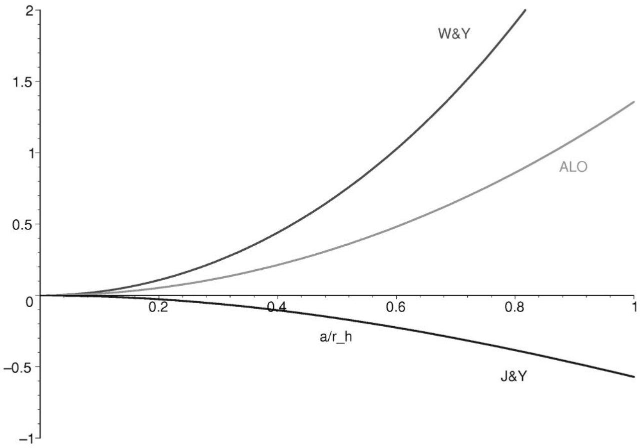

FIG. 2. Compare the coefficient of the pure spin-dependent term to the logarithmic correction. The curve denoted by J&Y [25] demonstrates that the entropy is decreased by the contribution from the s^2 term. Both the curve denoted by ALO [26] and that of ours denoted by W&Y illuminate that the spin-rotation coupling effect will increase the entropy of a Kerr black hole.

$$S/g_s=\frac{15+(-1)^{2s}}{16}\left(\frac{A_h}{48\pi\epsilon^2}+\frac{1}{45}\ln\frac{\Lambda}{\epsilon}\right)+\frac{3+(-1)^{2s}}{4}\;\frac{s^2}{12}\times\left[\frac{a^2-r_h^2}{r_h^2}+\frac{(r_h^2+a^2)^2}{ar_h^3}\arctan\left(\frac{a}{r_h}\right)\right]\ln\frac{\Lambda}{\epsilon}, \tag{35}$$

where $A_h=4\pi(r_h^2+a^2)$.

This result is very similar to that presented in Refs. [25] and [26]. In the scalar field case, it coincides with the result given by Ref. [20]. In the cases of $s=1/2$ (Dirac field), $s=1$ (Maxwell field), $s=3/2$ (Rarita-Schwinger field), and $s=2$ (linearized gravitational field), it has the same coefficients $s^2/6$ as that in Ref. [25]. But these coefficient terms have a different tendency because they differ by the terms in the square bracket. In the Rarita-Schwinger field ($s=3/2$) case, our result manifests that the spins will increase the entropies as pointed out by Ref. [26]. Again they differ by the coefficients of the quadratic term s^2. The expression $P=s^2a^2\cos^2\theta[\Delta-(r^2+a^2)-\Sigma]/\Sigma^3$ used in Refs. [25] and [26] but written in our notations, is probably the result that those authors had missed some terms in the process of their approximation. We point out that it should read $P=4s^2a^2\cos^2\theta[\Delta-(r^2+a^2)]/\Sigma^3$, and these coefficients presented in Refs. [25] and [26] are incorrect in the Kerr black hole case. The difference of the coefficients of the quadratic term s^2 (the pure spin-dependent term) between ours and those in Refs. [25] and [26] is compared in Fig. 2.

Case III: In the lukewarm case [29,30], the nonextreme Kerr–de Sitter spacetime is characterized by the condition $M^2=a^2\chi^2$. In the absence of rotation, the lukewarm solution reduces to de Sitter spacetime. When the cosmological constant vanishes, it degenerates to the extreme Kerr black hole. Obviously the lukewarm solution is a special case of what we have discussed in the above. In this case, the cosmological and outer black hole horizons are distinct, but the temperatures on both horizons are equal, so the cosmological and black hole event horizons are in thermal equilibrium. Just as the Kerr black hole case, here one need not assume local thermal equilibrium and he can still work with the original BWM to calculate the entropy of each horizon. Of course, the cutoff L cannot be so large as to close to the cosmological horizon. The final result of the entropy is still expressed by Eq. (32) with the special restriction (namely, $M^2=a^2\chi^2$) on the allowed range of parameter of the lukewarm solution. It should be noted that the same result can also be arrived at by using the thin-layer method. This issue can be easily understood as the brick wall entropy is mainly attributed to the degrees of freedom of the field in the near horizon region. As such we think the thin-layer method is more universal than the original BWM.

(f) We do not consider here the contribution from the super-radiant modes. As is well known, for rotating black holes in an asymptotically flat space, classical super-radiance effects occur only for bosonic but not fermionic fields [9], however the quantum analog of super-radiance does occur for both bosonic and fermionic fields [51]. It has been pointed out that the super-radiant modes have a contribution to the entropy of a rotating black hole [52]. In the Kerr black hole case, the effect of the super-radiant modes is to halve the leading contribution to the entropy of scalar and Dirac fields [12,16]. Their affect on the subleading term to the entropy for the nonextreme Kerr–de Sitter black hole is an interesting problem and deserves to be discussed elsewhere.

(g) We do not address here the renormalization of the divergence in the entropy of the nonextreme Kerr–de Sitter black hole. A generally accepted belief is that the matter field contributions to the entropy can be interpreted as one-loop corrections to classical Bekenstein-Hawking entropy. The divergence of the brick wall entropy has the same origin as the divergence of the one-loop effective action in quantum field theory in curved space. The leading quadratic divergence of the entropy calculated by the brick wall model can be absorbed by the renormalization of the Newton gravitational coupling constant in the one-loop effective action for matter and gravitation fields [53]. When gravity is described by a higher curvature effective action, the standard Bekenstein-Hawking result is only the leading contribution in the entropy, and there are still additional corrections from the higher curvature interaction [54]. On the other hand, quantum corrections in curved space are known to result in higher order curvature contributions to the Einstein-Hilbert action [55]. The subleading order logarithmic divergence in the entropy requires the introduction in the gravitational action of term quadratic in the curvature to be renormalized [56] (see also [19,34,57,58]). The standard renormalization of Newton gravitational coupling constant, the cosmological constant and other coefficients by the curvature squared (R^2) terms in the one-loop effective gravitational action [55] can remove all the divergent (quadratic and logarithmic) terms in the entropy [56], so the remaining quantity is finite.

In the case of nonextreme Kerr–de Sitter geometry, it is clear that the divergence coming from the integral in the dragged optical space can be completely removed by the above procedure, but it is unclear to us whether the logarithmic divergence arising from the effective potential including the quadratic spin terms needs further introduction of other interaction terms to be renormalized because this divergence explicitly contains quantum corrections to the brick wall entropy from the spin-rotation coupling interaction. Though the renormalization of the divergence in the brick wall entropy of the nonextreme Kerr–de Sitter black hole has not been discussed here, we hope the expression of the entropy obtained here can shed light on this subject because there is still little work on dealing with this problem in a rotating black hole spacetime.

V. CONCLUSIONS

In summary, we have studied the statistical entropies of the nonextreme Kerr–de Sitter black holes due to arbitrary spin fields, especially the subleading corrections to the black hole entropy arising from the coupling of the spins of particles with the rotation of the black holes. First, the null-tetrad in the Newman-Penrose formalism is introduced to decouple the Teukolsky master equations governing massless scalar, neutrino, electromagnetic, gravitino, and gravitational field perturbations of the Kerr–de Sitter space. Then, starting from the Teukolsky master equations we seek the total number of the modes of the fields by taking the WKB approximation. Last, the free energy and the quantum entropy of a nonextreme Kerr–de Sitter black hole due to arbitrary spin fields are calculated by the improved thin-layer brick wall model. It is shown that the subleading order contribution to the entropy is dependent on the square of three quantities, the spins of particles, the specific angular momentum of black holes, and the cosmological constant. The contribution of the spins of particles to the logarithmic terms of the entropy depends on the spin-rotation coupling effect and the effect of the cosmological constant. It should be noted that the final result also holds true in the lukewarm case where local thermal equilibrium need not be assumed.

In particular, we have carefully investigated the effect of a positive cosmological constant on the black hole entropy and shown that the correction from the effective potential is of the same order as that from the integral in the dragged optical space, and both of them cannot be discarded. However it should be noted that the correction from the "spin potential" W has been attributed to that of the vacuum surrounding the black holes and it has been neglected here. Possible new quantum effects related to the "sW" term in a Kerr–de Sitter black hole geometry may be a very interesting thing and deserve to be further investigated. It is also needed to extend this analysis to the Kerr-Newman–de Sitter black hole case.

ACKNOWLEDGMENTS

One of us (S.Q.W.) would like to thank Professor J. L. Jing for helpful discussions. He also thanks Professor N. Ohta from Osaka University for his comments when this work has been done. This project is partially supported by China Postdoctoral Science Foundation and K.C. Wong Education Foundation, Hong Kong. M.L.Y. has been partially supported by NSF of China under Grant No. 90103002. Parts of mathematical calculations were done when S.Q.W. stayed at the Institute of Particle Physics, Hua-Zhong Normal University, Wuhan. Some results by manual manipulations and their confirmations by MAPLE 7 have been done for several times while preparing this paper.

APPENDIX A: TEUKOLSKY-STAROBINSKY IDENTITIES

Teukolsky's master Equations (10) and (11) governing the perturbation of Kerr–de Sitter space with massless fields can be separated into the angular parts

$$\left[\sqrt{\Delta_\theta}\mathcal{L}^\dagger_{1-s}\sqrt{\Delta_\theta}\mathcal{L}_s - 2(2s-1)\times\left(\chi\omega a\cos\theta + (s-1)\frac{a^2}{l^2}\cos^2\theta\right) + \lambda_s\right]S_s = 0,$$

$$\left[\sqrt{\Delta_\theta}\mathcal{L}_{1-s}\sqrt{\Delta_\theta}\mathcal{L}^\dagger_s + 2(2s-1)\times\left(\chi\omega a\cos\theta - (s-1)\frac{a^2}{l^2}\cos^2\theta\right) + \lambda_s\right]S_{-s} = 0, \tag{A1}$$

and the radial parts

$$\left[\Delta_r\mathcal{D}_{1-s}\mathcal{D}^\dagger_0 + 2(2s-1)\left(i\chi\omega r - (s-1)\frac{r^2}{l^2}\right) - \lambda_s\right]\times(\Delta_r^s R_s) = 0,$$

$$\left[\Delta_r\mathcal{D}^\dagger_{1-s}\mathcal{D}_0 - 2(2s-1)\left(i\chi\omega r + (s-1)\frac{r^2}{l^2}\right) - \lambda_s\right]R_{-s} = 0. \tag{A2}$$

Here we only consider the radial equations. It is clear that $(\Delta_r^s R_s)$ and R_{-s} are proportional to complex conjugate functions. The exact relationship between these functions are called the famous Teukolsky-Starobinsky identities [39,40,45]

$$\Delta_r^s\mathcal{D}_0^{2s}R_{-s} = C_s(\Delta_r^s R_s), \qquad \Delta_r^s\mathcal{D}_0^{\dagger 2s}(\Delta_r^s R_s) = C_s^* R_{-s}, \tag{A3}$$

with the coefficients

$$|C_{1/2}|^2 = \lambda_{1/2},$$

$$|C_1|^2 = \lambda_1^2 - 4\chi^2\omega^2\alpha^2,$$

$$|C_{3/2}|^2 = (\lambda_{3/2}^2 + 4a^2/l^2)(\lambda_{3/2} + 1 - a^2/l^2) - 16\chi^2\omega^2(\lambda_{3/2}\alpha^2 + \alpha^4/l^2 - a^2) - 4M^2/l^2,$$

$$|C_2|^2=[\lambda_2(\lambda_2+2-2a^2/l^2)+12a^2/l^2]^2 -8\chi^2\omega^2\lambda_2[(5\lambda_2+6-6a^2/l^2+12\alpha^2/l^2)\alpha^2-12a^2] +144\chi^2\omega^2(\chi^2\omega^2\alpha^4+2a^2\alpha^2/l^2+M^2), \tag{A4}$$

where $\alpha^2=a^2-ma/\omega$. The coefficient $|C_2|^2$ corrects the previous results [46], while the appearance of $|C_{3/2}|^2$ is the first time, to our knowledge.

APPENDIX B: INTEGRALS IN TERMS OF THIN-LAYER BRICK WALL MODEL

Distinguished from the original BWM, the thin-layer BWM suggests that the entropy of a black hole with two horizons mainly comes from a very thin layer in the vicinity of the horizon where exists a local thermal equilibrium. Just as the original BWM, it also imposes a small ultraviolet cutoff ε such that

$$\Psi(x)=0 \quad \text{for } r\leqslant r_h+\varepsilon, \tag{B1}$$

where r_h denotes one coordinate of the two horizons of the nonextreme Kerr–de Sitter black hole. In this paper, it represents the outer black hole event horizon or the cosmological horizon, satisfying the horizon equation

$$\Delta_{r_h}=(r_h^2+a^2)\left(1-\frac{r_h^2}{l^2}\right)-2Mr_h=0. \tag{B2}$$

To remove the infrared divergence, it however introduces another cutoff parameter—an arbitrary big integer N such that

$$\Psi(x)=0 \quad \text{for } r\geqslant r_h+N\varepsilon. \tag{B3}$$

Suppose that the quantum field is rotating with the angular velocity $\Omega_h=a/(r_h^2+a^2)$ in the thin layer near the horizon of the Kerr–de Sitter black hole, we may expand Δ_r close to the event horizon r_h as

$$\Delta_r=\Delta'_{r_h}(r-r_h)+\frac{1}{2}\Delta''_{r_h}(r-r_h)^2+\cdots, \tag{B4}$$

and then expand three quantities $\tilde{g}_{tt}$, P, and W in terms of the surface gravity $\kappa_h=\Delta'_{r_h}/(2\chi(r_h^2+a^2))=2\pi\Delta'_{r_h}/(\chi^2A_h)$ as follows:

$$\tilde{g}_{tt}=\frac{\Delta_\theta a^2\sin^2\theta(r^2-r_h^2)^2-\Delta_r\Sigma_h^2}{\chi^2(r_h^2+a^2)^2\Sigma} \approx\frac{-2\kappa_h\Sigma_h(r-r_h)}{\chi(r_h^2+a^2)} \times\left[1-\left(\frac{2r_h}{\Sigma_h}-\frac{\Delta''_{r_h}}{2\Delta'_{r_h}}+\frac{4r_h^2\Delta_\theta a^2\sin^2\theta}{\Delta'_{r_h}\Sigma_h^2}\right)(r-r_h)\right]+\cdots,$$

$$P=\frac{4s^2+2}{l^2}+\frac{4s^2a^2\cos^2\theta}{\Sigma^3} \times\left[\Delta_r-\Delta_\theta(r^2+a^2)+\frac{a^2\sin^2\theta}{l^2}\Sigma\right] \approx\frac{4s^2+2}{l^2}+\frac{4s^2a^2\cos^2\theta}{\Sigma_h^3}\left[\frac{a^2\sin^2\theta}{l^2}\Sigma_h-\Delta_\theta(r_h^2+a^2)\right]+\cdots,$$

$$W=\frac{-a\cos\theta}{\chi(r_h^2+a^2)\Sigma^2}\left\{\left[\chi\Sigma+2a^2\sin^2\theta\left(1-\frac{r^2}{l^2}\right)\right] \times(r^2-r_h^2)+2\Delta_r\Sigma_h\right\} \approx\frac{-4\kappa_h a\cos\theta}{\Sigma_h}\left\{1+\frac{r_h}{\Delta'_{r_h}}\left[\chi+\frac{2a^2\sin^2\theta}{\Sigma_h}\left(1-\frac{r_h^2}{l^2}\right)\right]\right\} \times(r-r_h)+\cdots, \tag{B5}$$

where $\Sigma_h=r_h^2+a^2\cos^2\theta$.

Expanding the integrands in the four integrals $I_1\sim I_4$ defined in Eq. (26)

$$\frac{\sqrt{-g}}{\tilde{g}_{tt}^2}\approx\frac{(r_h^2+a^2)^2\sin\theta}{4\kappa_h^2\Sigma_h}\left[\frac{1}{(r-r_h)^2}+\left(\frac{6r_h}{\Sigma_h}-\frac{\Delta''_{r_h}}{\Delta'_{r_h}}+\frac{8r_h^2\Delta_\theta a^2\sin^2\theta}{\Delta'_{r_h}\Sigma_h^2}\right)\frac{1}{r-r_h}\right]+\cdots \approx\frac{(r_h^2+a^2)\sin\theta}{4\chi\kappa_h^3\Sigma_h}\left[\frac{\Delta'_{r_h}}{2(r-r_h)^2}+\left(\frac{3r_h\Delta'_{r_h}}{\Sigma_h}-\frac{1}{2}\Delta''_{r_h}+\frac{4r_h^2\Delta_\theta a^2\sin^2\theta}{\Sigma_h^2}\right)\frac{1}{r-r_h}\right]+\cdots,$$

$$\frac{\sqrt{-g}}{\tilde{g}_{tt}}P=\frac{-(r_h^2+a^2)\sin\theta}{2\chi\kappa_h(r-r_h)}\left\{\frac{4s^2+2}{l^2}+4s^2a^2\cos^2\theta\left[\frac{a^2\sin^2\theta}{l^2\Sigma_h^2}-\frac{\Delta_\theta(r_h^2+a^2)}{\Sigma_h^3}\right]\right\}+\cdots, \tag{B6}$$

$$\frac{\sqrt{-g}}{\tilde{g}_{tt}^2}W^2=\frac{4(r_h^2+a^2)^2a^2\sin\theta\cos^2\theta}{\Sigma_h^3}\left\{1+\frac{r_h}{\Delta'_{r_h}}\left[\chi+\frac{2a^2\sin^2\theta}{\Sigma_h}\left(1-\frac{r_h^2}{l^2}\right)\right]\right\}^2+\cdots,$$

$$\frac{\sqrt{-g}}{\tilde{g}_{tt}}\frac{\Delta''_r}{4\Sigma}W=\frac{\Delta''_{r_h}(r_h^2+a^2)a\sin\theta\cos\theta}{2\chi\Sigma_h^2}\left\{1+\frac{r_h}{\Delta'_{r_h}}\left[\chi+\frac{2a^2\sin^2\theta}{\Sigma_h}\left(1-\frac{r_h^2}{l^2}\right)\right]\right\}+\cdots,$$

and carrying out the integrals with respect to θ and r, we finally arrive at

$$\begin{aligned}
I_1&=\frac{1}{\chi\kappa_h^3}\left[\frac{r_h^2+a^2}{\eta^2}\frac{N-1}{N}\frac{\Sigma_h}{ar_h}\arctan\left(\frac{a}{r_h}\right)+\left(1-\frac{3r_h^2+a^2}{2l^2}\right)\ln N\right]\\
&=\frac{2}{\chi\kappa_h^3}\left[\frac{15(r_h^2+a^2)}{4\epsilon^2}+\left(1-\frac{3r_h^2+a^2}{2l^2}\right)\ln\frac{\Lambda}{\epsilon}\right],\\
I_2&=\frac{1}{4\chi\kappa_h}\left\{-\frac{4+8s^2}{l^2}(r_h^2+a^2)+s^2\left[\frac{a^2-r_h^2}{r_h^2}+\frac{9r_h^2+7a^2}{l^2}+\left(\frac{r_h^2+a^2}{r_h^2}-\frac{9r_h^2+a^2}{l^2}\right)\frac{r_h^2+a^2}{ar_h}\arctan\left(\frac{a}{r_h}\right)\right]\right\}\ln N\\
&=\frac{1}{2\chi\kappa_h}\left\{-\frac{4(r_h^2+a^2)}{l^2}+s^2\left[\frac{a^2-r_h^2}{r_h^2}+\frac{r_h^2-a^2}{l^2}+\left(\frac{r_h^2+a^2}{r_h^2}-\frac{9r_h^2+a^2}{l^2}\right)\frac{r_h^2+a^2}{ar_h}\arctan\left(\frac{a}{r_h}\right)\right]\right\}\ln\frac{\Lambda}{\epsilon},\\
I_3&\sim O(\epsilon),\qquad I_4=0,
\end{aligned}\tag{B7}$$

where $A_h=4\pi(r_h^2+a^2)/\chi$ is the horizon area. In the last step, we have replaced the ultraviolet cutoff $\varepsilon=\eta^2\Delta'_{r_h}/(4\Sigma_h)$ by the proper distance η from the horizon to the inner brick wall $\eta=\int_{r_h}^{r_h+\varepsilon}\sqrt{g_{rr}}dr\approx2(\varepsilon\Sigma_h/\Delta'_{r_h})^{1/2}$. To be comparable with Refs. [20,25] and [26], the new infrared cutoff Λ and ultraviolet cutoff ϵ in the above equation (B7) are defined by

$$N=\Lambda^2/\epsilon^2,\qquad \eta^2=\frac{2\epsilon^2}{15}\frac{N-1}{N}\frac{\Sigma_h}{ar_h}\arctan\left(\frac{a}{r_h}\right).\tag{B8}$$

For large N and small a, Eq. (B7) implies $\eta^2=2\epsilon^2/15$, which is used in the context.

[1] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973); **9**, 3294 (1974).

[2] S. W. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).

[3] W. H. Zurek and K. S. Thorne, Phys. Rev. Lett. **54**, 2171 (1985).

[4] L. Bombelli, R. Koul, J. Lee, and R. Sorkin, Phys. Rev. D **34**, 373 (1986).

[5] V. P. Frolov and I. D. Novikov, Phys. Rev. D **48**, 4545 (1993); V. P. Frolov, Phys. Rev. Lett. **74**, 3319 (1995); V. P. Frolov, D. V. Fursaev, and A. I. Zelnikov, Nucl. Phys. **B486**, 339 (1997); V. P. Frolov and D. V. Fursaev, Phys. Rev. D **61**, 064010 (2000).

[6] J. D. Bekenstein, gr-qc/9409015.

[7] G. 't Hooft, Nucl. Phys. **B256**, 727 (1985).

[8] E. Newman and R. Penrose, J. Math. Phys. **3**, 566 (1962).

[9] S. Chandrasekhar, *The Mathematical Theory of Black Holes* (Oxford University Press, New York, 1983).

[10] A. Ghosh and P. Mitra, Phys. Rev. Lett. **73**, 2521 (1994); Phys. Lett. B **357**, 295 (1995).

[11] M. H. Lee and J. K. Kim, Phys. Rev. D **54**, 3904 (1996); Phys. Lett. A **212**, 323 (1996).

[12] J. Ho, W. T. Kim, and Y. J. Park, Class. Quantum Grav. **14**, 2617 (1997).

[13] Y. G. Shen and D. M. Chen, Gen. Relativ. Gravit. **32**, 2269 (2000); Int. J. Mod. Phys. D **10**, 539 (2001); C. J. Gao and Y. G. Shen, Class. Quantum Grav. **19**, 4933 (2002); Chin. Phys. Lett. **19**, 474 (2002); Y. H. Wei, Y. C. Wang, and Z. Zhao, Phys. Rev. D **65**, 124 023 (2002).

[14] Y. G. Shen, Gen. Relativ. Gravit. **32**, 1647 (2000); Phys. Lett. A **236**, 234 (2000); Mod. Phys. Lett. A **15**, 1901 (2000).

[15] G. Cognola and P. Lecca, Phys. Rev. D **57**, 1108 (1997); J. L. Jing, Int. J. Theor. Phys. **39**, 1687 (2000); Chin. Phys. **9**, 389 (2000).

[16] W. B. Liu and Z. Zhao, Phys. Rev. D **61**, 063003 (2000); Y. G. Shen and Z. Y. Chen, Int. J. Theor. Phys. **40**, 1009 (2001).

[17] S. Q. Wu and X. Cai, Chin. Phys. Lett. **18**, 485 (2001); Gen. Relativ. Gravit. **33**, 1185 (2001); *ibid.* **34**, 557, 605 (2002).

[18] M. Carmeli and M. Kaye, Ann. Phys. (N.Y.) **103**, 97 (1977); M. Carmeli, *Classical Fields: General Relativity and Gauge Theory*, (Wiley, New York, 1982); C. Gonzalez, L. Herrera, and J. Jimenez, J. Math. Phys. **20**, 837 (1979).

[19] S. N. Solodukhin, Phys. Rev. D **51**, 609 (1995); **51**, 618 (1995); **52**, 7046 (1995); **56**, 4968 (1997).

[20] R. B. Mann and S. N. Solodukhin, Phys. Rev. D **54**, 3932 (1996).

[21] G. Cognola, Phys. Rev. D **57**, 6292 (1998).

[22] J. L. Jing and M. L. Yan, Phys. Rev. D **60**, 084015 (1999); **61**, 044016 (2000); M. H. Lee and J. K. Kim, hep-th/9604130.

[23] Z. H. Li, Int. J. Theor. Phys. **39**, 253 (2000); Nuovo Cimento B **116**, 965 (2001); Y. G. Shen and C. J. Gao, Chin. J. Astron. Astrophys. **1**, 357 (2001); Gen. Relativ. Gravit. **34**, 1193 (2002); Y. G. Shen, Phys. Lett. B **537**, 187 (2002); Sci. China, Ser. A: Math., Phys., Astron. **45**, 400 (2002); C. J. Gao and Y. G. Shen, Phys. Rev. D **65**, 084043 (2002).

[24] Z. H. Li, Phys. Rev. D **62**, 024001 (2000).

[25] J. L. Jing and M. L. Yan, Phys. Rev. D **63**, 084028 (2001); **64**, 064015 (2001); J. L. Jing, Chin. Phys. Lett. **20**, 459 (2003).

[26] A. López-Ortega, Gen. Relativ. Gravit. **35**, 59 (2003).

[27] C. J. Gao and W. B. Liu, Int. J. Theor. Phys. **39**, 2221 (2000); W. B. Liu and Z. Zhao, Int. J. Mod. Phys. A **16**, 3793 (2001); Chin. Phys. Lett. **18**, 310 (2001).

[28] X. Li and Z. Zhao, Phys. Rev. D **62**, 104001 (2000); F. He, Z. Zhao, and S. W. Kim, *ibid.* **64**, 044025 (2001).

[29] F. Mellor and I. Moss, Class. Quantum Grav. **6**, 1379 (1989).

[30] I. S. Booth and R. B. Mann, Phys. Rev. Lett. **81**, 5052 (1998); Nucl. Phys. B **539**, 267 (1999).

[31] A. G. Reiss *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).

[32] J. P. Ostriker and P. J. Steinhardt, Nature (London) **377**, 600 (1995).

[33] M. H. Dehghani and R. B. Mann, Phys. Rev. D **64**, 044003 (2001); M. H. Dehghani, *ibid.* **65**, 104003 (2002); **65**, 104030 (2002).

[34] E. Winstanley, Phys. Rev. D **63**, 084013 (2001).

[35] X. H. Ge and Y. G. Shen, Chin. Phys. Lett. **19**, 1051 (2002); Chin. Astron. Astrophys. **27**, 140 (2003).

[36] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977); B. Carter, Commun. Math. Phys. **10**, 280 (1968); **17**, 233 (1970).

[37] U. Khanal, Phys. Rev. D **28**, 1291 (1983).

[38] H. Suzuki, E. Takasugi, and H. Umetsu, Prog. Theor. Phys. **100**, 491 (1998); **102**, 253 (1999).

[39] S. A. Teukolsky, Phys. Rev. Lett. **29**, 1114 (1972); Astrophys. J. **185**, 635 (1973); W. H. Press and S. A. Teukolsky, *ibid.* **185**, 649 (1973); S. A. Teukolsky and W. H. Press, *ibid.* **193**, 443 (1974).

[40] G. F. Torres del Castillo, J. Math. Phys. **29**, 2078 (1988); **30**, 446 (1989); **30**, 1323 (1989).

[41] E. W. Leaver, J. Math. Phys. **27**, 1238 (1986); S. Mano, H. Suzuki, and E. Takasugi, Prog. Theor. Phys. **95**, 1079 (1996); **96**, 549 (1996).

[42] S. Q. Wu and X. Cai, J. Math. Phys. **40**, 4538 (1999); **44**, 1084 (2003).

[43] H. Suzuki, E. Takasugi, and H. Umetsu, Prog. Theor. Phys. **103**, 723 (2000); H. Umetsu, *ibid.* **104**, 743 (2000).

[44] T. Masuda and H. Suzuki, J. Math. Phys. **38**, 3669 (1997).

[45] E. G. Kalnis, W. Miller, Jr., and G. C. Williams, J. Math. Phys. **30**, 2925 (1989); S. Mano and E. Takasugi, Prog. Theor. Phys. **97**, 213 (1997).

[46] U. Khanal, Phys. Rev. D **32**, 879 (1985); C. M. Chambers and I. G. Moss, Class. Quantum Grav. **11**, 1035 (1994).

[47] S. P. de Alwis and N. Ohta, Phys. Rev. D **52**, 3529 (1995).

[48] S. Q. Wu and X. Cai, gr-qc/9907054.

[49] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon Press, London, 1976); R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Clarendon, Oxford, 1934).

[50] S. Mukohyama and W. Israel, Phys. Rev. D **58**, 104005 (1998).

[51] W. G. Unruh, Phys. Rev. D **10**, 3194 (1974); E. Winstanley, *ibid.* **64**, 104010 (2001).

[52] S. W. Kim, W. T. Kim, Y. J. Park, and H. Shin, Phys. Lett. B **392**, 311 (1997); J. Ho and G. Kang, *ibid.* **445**, 27 (1998).

[53] L. Susskind and J. Uglum, Phys. Rev. D **50**, 2700 (1994); J. L. F. Barbón and R. Emparan, *ibid.* **52**, 4527 (1995); F. Larsen and F. Wilczek, Nucl. Phys. **B458**, 249 (1996).

[54] R. M. Wald, Phys. Rev. D **48**, 3427 (1993); T. Jacobson and G. Kang, *ibid.* **49**, 6587 (1994).

[55] N. D. Brill and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, New York, 1982).

[56] D. V. Fursaev and S. N. Solodukhin, Phys. Lett. B **365**, 51 (1996); T. Shimomura, *ibid.* **480**, 207 (2000); V. P. Frolov and D. V. Fursaev, Class. Quantum Grav. **15**, 2041 (1998).

[57] J. G. Demers, R. Lafrance, and R. C. Myers, Phys. Rev. D **52**, 2245 (1995).

[58] S. P. Kim, S. K. Kim, K. S. Soh, and J. H. Yee, Phys. Rev. D **55**, 2159 (1997); X. Li, *ibid.* **65**, 084005 (2002).